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GENERAL BPS BLACK HOLES IN FIVE DIMENSIONS

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Abstract

We construct general static black hole configuration for the theory of $N = 2$, $d = 5$ supergravity coupled to an arbitrary number of Abelian vector multiplets. The underlying *very special geometry* structure plays a major role in this construction. From the viewpoint of M-theory compactified on a Calabi-Yau threefold, these black holes are identified with BPS winding states of the membrane around 2-cycles of the Calabi-Yau threefold, and thus are of importance in the probing of the phase transitions in the moduli space of M-theory compactified on a Calabi-Yau threefold.

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1 Introduction

Recently progress has been made in the study of extreme solutions of $N = 2$ supergravity in four and five dimensions. These solutions have a rich structure in contrast to those of $N = 4, 8$ supergravity theories, where the dynamics is severely restricted by supersymmetry. Namely, black holes of $N = 4, 8$ supersymmetric theories do not receive quantum corrections whereas solutions of $N = 2$ supergravity alter significantly when quantum corrections are taken into account. The starting point in the understanding of non-singular static $N = 2$ black holes came with the realization that their Bekenstein-Hawking entropy, $\mathbf{S}_{BH}$, [1] can be obtained by extremizing the central charge of the underlying $N = 2$ superalgebra with respect to the moduli [2],[3],[5]. The entropy of these black holes was found to satisfy the following universal formula

$$\mathbf{S}_{BH} \sim |Z_{fix}|^{\alpha} \tag{1}$$

where $\alpha = 2, \frac{3}{2}$ for $d = 4, 5$ respectively and Z_{fix} is the extremal value of the central charge.

Moreover, it was shown (for the 4-dimensional case), that the scalar fields follow attractor equations with fixed points on the horizon [2]. These fixed values are independent of the values of the scalar fields at infinity (moduli) and are determined only in terms of conserved charges and topological data. If one assumes that the scalar fields take these fixed values throughout the entire space-time, one obtains in this way the so-called double extreme black holes [4].

The above results are covariantly formulated in terms of the underlying special geometry [6]. For instance, the equations which give the values of the scalar fields near the horizon, the so-called stabilisation equations for the double-extreme black holes are given by [4]

$$i(\bar{Z}L^I - Z\bar{L}^I) = p^I \qquad , \qquad i(\bar{Z}M_I - Z\bar{M}_I) = q_I, \tag{2}$$

where (L^I, M_I) are the covariantly holomorphic section of $N = 2$ supergravity [6], p^I and q_I are, respectively, integer valued magnetic and electric charges. By means of the equations (2), one can, in principle compute the classical entropy of static $N = 2$ black holes [7], [8] as well as to incorporate quantum corrections [9], [10].[b]. For example, in type II string compactifications, the entropy depends on the topological data of the Calabi-Yau spaces, like the intersection numbers, the Euler number and the rational worldsheet instanton numbers [9].

Later extreme solutions, with non-constant values for the moduli, were derived, by solving for the equations of motion, for the axion-free STU model with cubic prepotential, and for supergravity models based on quadratic prepotentials [11],[8]. It was then shown in [12] that for general static extreme $N = 2$ black holes, the solutions are completely specified by the Kähler potential of the underlying special geometry where the imaginary part of the

[b] The quantum corrections are encoded in the M_I part of the covariantly holomorphic section

holomorphic sections are given in terms of a set of constrained harmonic functions. These constraints correspond to asymptotic flatness and the vanishing of the Kähler connection. Moreover, it was shown in [13] that the metric for stationary, but non-static solutions depends also on the $U(1)$ Kähler connection. Depending on the choices for the harmonic functions and on the considered prepotentials, one gets for example non-static rotating $N=2$ black holes, $N=2$ Taub-NUT spaces or $N=2$ Eguchi-Hanson like instantons.

More concretely, in four dimensions, it is very well known from the work of Tod [14], that the most general form of the metric admitting supersymmetries can be written in the IWP [15] metric form

$$ds^2 = -e^{2U}(dt + \omega_m dx^m)^2 + e^{-2U} dx^m dx^m, \tag{3}$$

and ω_m is defined by

$$\vec{\nabla} \times \vec{\omega} = -\frac{i}{(V\bar{V})^2}(\bar{V}\vec{\nabla}V - V\vec{\nabla}\bar{V}) \tag{4}$$

where $e^{2U} = V\bar{V}$ and V is the inverse of a harmonic function.

For $N=2$ supergravity models with vector multiplets, the metric is of the form (3), with

$$\begin{aligned}
e^{-2U} &= e^{-K} \equiv i(\bar{X}^I F_I - X^I \bar{F}_I) \ , \\
\tfrac{1}{2}e^{2U}\epsilon_{mnp}\partial_n\omega_p &= Q_m \equiv \tfrac{1}{2}e^K(\bar{F}_I\partial_m X^I - \bar{X}^I\partial_m F_I + c.c.) \\
&= \tfrac{1}{2}e^{2U}(H_I\partial_m\tilde{H}^I - \tilde{H}^I\partial_m H_I)
\end{aligned} \tag{5}$$

where the equations which define the moduli fields as well the electric and magnetic fields strengths are given by

$$i(X^I - \bar{X}^I) = \tilde{H}^I(x^\mu) \quad , \quad i(F_I - \bar{F}_I) = H_I(x^\mu) \tag{6}$$

$$F^I_{mn} = \frac{1}{2}\epsilon_{mnp}\partial_p\tilde{H}^I \quad , \quad G_{I\,mn} = \frac{1}{2}\epsilon_{mnp}\partial_p H_I \tag{7}$$

here (X^I, F_I) is the symplectic holomorphic section, and $(H_I, \tilde{H}^I)$ are harmonic functions. For vanishing Kähler connection ($Q_m = 0$), one obtains static solutions.

Using the explicit static black hole solutions, one can calculate the entropy and the value of the scalar fields near the horizon. It can be easily demonstrated that the entropy is given in terms of the extremum of the central charge and that the equations (6) reduce to those of (2) near the horizon [12].

In five dimensions, supersymmetric black holes were constructed for the case of pure $N=2$ supergravity in [20]. The metric in this case is of the Tanghelini form [21],[22]. In the context of supergravity with vector multiplets, double-extreme black holes were constructed and their entropy was calculated in terms of the extremised central charge in

[25]. Moreover, in [25], the Strominger-Vafa black hole [26] was reproduced as a double-extreme black hole of an $N = 2$ supergravity model with one vector multiplet. Also the rotating black hole solution of [27] was embedded into an N=2 supergravity model with one vector multiplet, and was shown to admit unbroken supersymmetry [28]. Five dimensional static and rotating black holes were also constructed in [29] and [30] in the context of heterotic and type II string toroidal compactifications.

In spite of lots of work on the five dimensional black holes, general supersymmetric solutions with non-constant values of the moduli, surprisingly, have not been constructed yet. It is our purpose in this paper to find explicit black hole solutions of $N = 2$ $d = 5$ supergravity coupled to an arbitrary number of vector multiplets. Here we will only concentrate on the electrically charged static solution and leave the rotating solutions as well as the magnetic black string solutions for a separate publication. In five dimensions, the couplings of $N = 2$ supergravity to Abelian vector multiplet, is based on the structure of very special geometry and thus one expects that this geometric structure should play a major role in the study of black hole solutions similar to that played by special geometry in four dimensions.

This work is organised as follows. In the next section, we briefly review the structure of $N = 2$ supergravity in $d = 5$ and collect some formulae and expressions which will be relevant for our later discussion. In section three, we will give the static black hole solutions and verify that they admit unbroken supersymmetry by solving for the supersymmetry transformation rules for the gravitino and the gauginos in a bosonic background. We will also study the behaviour of our solutions near the horizon and demonstrate that the values of the moduli take fixed values which extremise the central charge. We also derive the expression of the entropy in terms of the extremised central charge, and give the general form of the double-extreme black hole solutions. The last section contains a summary of our results and a discussion on the relevance of the black hole solutions to the analysis of topological phase transitions of M-theory on a Calabi-Yau 3-folds.

2 $d = 5$ $N = 2$ Supergravity and Very Special Geometry

The theory of $N = 2$ supergravity theory coupled to an arbitrary number n of Maxwell's supermultiplets was first considered in [19]. Recent discussions of the bosonic part (our main concern in this work) in terms of very special geometry is given in [31]. Also, the construction of $N = 2$ supergravity as a compactification of $d = 11$ supergravity down to five dimensions on Calabi-Yau 3-folds was discussed more recently in [32].

In the original analysis of [19], it was established that the scalar fields of the vector multiplets parametrise a riemannian space. Those which are homogeneous symmetric spaces take the form,

$$\mathcal{M} = \frac{\mathrm{Str}_0(J)}{\mathrm{Aut}(J)}, \tag{8}$$

where $\mathrm{Str}_0(J)$ is the reduced structure group of a formally real unital Jordan Algebra of degree three, $\mathrm{Aut}(J)$ is its automorphism group. The classification of homogeneous symmetric spaces was then reduced to that of Jordan algebras of degree three. Moreover, the scalar manifold can be regarded as a hyperspace, with vanishing second fundamental form of an $(n+1)$-dimensional riemannian space $\mathcal{G}$ whose coordinates X are in correspondence with the vector multiplets including that of the graviphoton. The equation of the hypersurface is $\mathcal{V} = 1$, where $\mathcal{V}$, the prepotential, is a homogeneous cubic polynomial in the coordinates of $\mathcal{G}$,

$$\mathcal{V} = \frac{1}{6} C_{IJK} X^I X^J X^K \ . \tag{9}$$

Non-simple Jordan algebras of degree three are of the form $R \oplus \Sigma_n$, where Σ_n is the Jordan algebra associated with a quadratic form. The corresponding symmetric scalar manifold are the form

$$\mathcal{M} = SO(1,1) \times \frac{SO(n-1,1)}{SO(n-1)}. \tag{10}$$

In this case, $\mathcal{V}$ is factorisable into a linear times a quadratic form in $(n-1)$ scalars, which for the positivity of the kinetic terms in the Lagrangian, must have a Minkowski metric. For simple Jordan algebras, one obtains four sporadic locally symmetric spaces related to the four simple unital formally real Jordan algebras over the four division algebras of real, complex, quaternions and octanions. For more details we refer the reader to [19].

For M-theory compactifications on a Calabi-Yau threefold with Hodge numbers $h_{(1,1)}$, $h_{(2,1)}$, and intersection numbers C_{IJK}, $(I, J, K = 1, \cdots h_{(1,1)})$, $\mathcal{V}$ represents the intersection form of the Calabi-Yau threefold related to the overall volume of the Calabi-Yau threefold and belongs to the so-called universal hypermultiplet. Also, in this picture, X^I correspond to the size of the 2-cycles of the Calabi-Yau threefold. The massless spectrum of the theory contains $h_{(1,1)} - 1$ vector multiplets with real scalar components defined by the moduli at unit volume. Including the graviphoton, the theory has $h_{(1,1)}$ vector bosons. In addition to the universal hypermultiplet present in any Calabi-Yau compactification, the theory also contains $h_{(2,1)}$ hypermultiplets.[c]

We now turn to discuss the effective $N = 2$ supersymmetric Lagrangian describing the coupling of vector multiplets to supergravity in five dimensions. This Lagrangian is determined entirely in terms of a cubic prepotential (9), which defines very special geometry [31]. For our purpose, we only need to display the bosonic part of the Lagrangian and the supersymmetry transformation laws for the gravitino and the gauginos. The bosonic

[c]Briefly, in the 11-dimensional theory, we have the metric $G_{\hat{\mu}\hat{\nu}}$ and a three-form gauge field $\mathcal{A}_{\hat{\mu}\hat{\nu}\hat{\rho}}$. If we split $\hat{\mu}$ into $(\mu, a, \bar{a})$, with $(\mu = 1, \cdots, 5)$ the space-time indices and $(a, \bar{a})$ represent the three internal complex dimensions of the Calabi-Yau, then the $h_{(1,1)} - 1$ scalar moduli correspond to $G_{a\bar{b}}$ defined at unit volume $(detG_{a\bar{b}} = 1.)$ The vector bosons correspond to the $h_{(1,1)}$ space-time one-forms $\mathcal{A}_{\mu a\bar{a}}$. The universal hypermultiplet contains $(\mathcal{V}, \mathcal{A}_{\mu\nu\rho}, \mathcal{A}_{abc} = \epsilon_{abc} C)$, the $h_{(2,1)}$ hypermultiplets correspond to $(G_{ab}, \mathcal{A}_{ab\bar{c}})$

action is

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}G_{IJ}F_{\mu\nu}{}^{I}F^{\mu\nu J} - \frac{1}{2}g_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + \frac{e^{-1}}{48}\epsilon^{\mu\nu\rho\sigma\lambda}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^k_\lambda \tag{11}$$

where R is the scalar curvature, $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ is the Maxwell field-strength tensor and $e = \sqrt{-g}$ is the determinant of the Fünfbein $e^{\underline{\nu}}_{\mu}$. [d]

The fields $X^I = X^I(\phi)$ are the special coordinates satisfying

$$X^I X_I = 1, \qquad \frac{1}{6}C_{IJK}X^I X^J X^K = 1 \tag{12}$$

where, X_I, the dual coordinate is defined by,

$$X_I = \frac{1}{6}C_{IJK}X^J X^K. \tag{13}$$

The moduli- dependent gauge coupling metric is related to the prepotential (9) via the relation

$$G_{IJ} = -\frac{1}{2}\frac{\partial}{\partial X^I}\frac{\partial}{\partial X^J}(\ln\mathcal{V})|_{\mathcal{V}=1} \tag{14}$$

The metric g_{ij} is given by

$$g_{ij} = G_{IJ}\partial_i X^I \partial_j X^J|_{\mathcal{V}=1}; \qquad (\partial_i \equiv \frac{\partial}{\partial\phi^i}) \tag{15}$$

The supersymmetry transformation laws for the Fermi fields in a bosonic background are given by [19]

$$\begin{aligned} \delta\psi_\mu &= \mathcal{D}_\mu\epsilon + \frac{i}{8}X_I\Big(\Gamma_\mu{}^{\nu\rho} - 4\delta_\mu{}^\nu\Gamma^\rho\Big)F_{\nu\rho}{}^I\epsilon, \\ \delta\lambda_i &= \frac{3}{8}\partial_i X_I\Gamma^{\mu\nu}\epsilon F^I_{\mu\nu} - \frac{i}{2}g_{ij}\Gamma^\mu\partial_\mu\phi^j\epsilon, \\ &. \end{aligned} \tag{16}$$

where ϵ is the supersymmetry parameter and

$$\mathcal{D}_\mu = \partial_\mu + \frac{1}{4}\omega_{\mu\underline{\nu}\underline{\rho}}\Gamma^{\underline{\nu}}\Gamma^{\underline{\rho}} \tag{17}$$

Here, $\omega_{\mu\underline{\nu}\underline{\rho}}$ is the spin connection, Γ^ν are Dirac matrices and

$$\Gamma^{\nu_1\nu_2\cdots\nu_n} = \frac{1}{n!}\Gamma^{[\nu_1}\Gamma^{\nu_2}\cdots\Gamma^{\nu_n]}. \tag{18}$$

[d] In this paper, we will use for the metric the signature $(-++++)$ and for the indices we take: $\mu,\nu.. = 0,1,2,3,4$, whereas $m,n,\cdots = 1,2,3,4$. We denote the underlined indices $\underline{\mu},\underline{\nu}$ as flat ones. Antisymmetrized indices are defined by: $[\mu\nu] = \frac{1}{2}(\mu\nu - \nu\mu)$.

3 Extreme Black Hole Solutions

In this paper, we are interested in electrically charged BPS black hole solutions, *i.e* configurations that break half of the supersymmetry. In the context of M-theory compactification on a Calabi-Yau, these correspond to BPS winding states of the membrane around 2-cycles of the Calabi-Yau threefold. Magnetically charged (string) solutions, which in the M-theory picture correspond to winding BPS states of the five-brane around the 4-cycles, together with general stationary electrically charged solutions will be discussed elsewhere [34].

Our ansatz for the BPS black hole solution is

$$\begin{aligned} ds^2 &= -e^{-4U}dt^2 + e^{2U}(d\vec{x})^2 \\ G_{IJ}F^I_{0m} &= \frac{1}{2}e^{-4U}\partial_m H_J, \qquad \tilde{X}_I = e^{2U}X_I = \frac{1}{3}H_I \\ e^{3U} &= \frac{1}{6}C_{IJK}\tilde{X}^I\tilde{X}^J\tilde{X}^K = \tilde{X}_I\tilde{X}^I, \qquad \tilde{X}^I = e^U X^I \\ H_I &= h_I + \frac{q_I}{r^2} \end{aligned} \tag{19}$$

where h_I are constants related to the values of the scalars at infinity and q_I are electric charges.

Before we proceed to show that the above configuration is indeed a BPS one, we will derive some relations which will be useful for our purposes. First if one differentiate (12) with respect to the scalar fields, keeping in mind that C_{IJK} is a constant symmetric tensor we obtain

$$\partial_i X_I = \frac{1}{3}C_{IJK}\partial_i X^J X^K \qquad X^I\partial_i X_I = X_I\partial_i X^I = 0. \tag{20}$$

Moreover, the gauge coupling metric G_{IJ} can be expressed in terms of the special coordinates by

$$G_{IJ} = -\frac{1}{2}C_{IJK}X^K + \frac{9}{2}X_I X_J \tag{21}$$

In order to relate the derivative of the special coordinate to that of its dual, we use (21) together with (20), this gives the following expression

$$\partial_i X_I = -\frac{2}{3}G_{IJ}\partial_i X^J. \tag{22}$$

Finally it is very useful to find an expression for the graviphoton field-strength ($X_I F^I$), in our Ansatz. In order to do this, we note that from (19) we obtain

$$e^{2U} = \frac{1}{3}H_I X^I. \tag{23}$$

Differentiating both sides of the above equation and using the second equation on (20) yields

$$\partial_m(e^{2U}) = \frac{1}{3}\partial_m(H_I)X^I \tag{24}$$

Moreover, using (21) and (13) we obtain the following relation

$$X_I = \frac{2}{3} G_{IJ} X^J, \tag{25}$$

therefore,

$$X_I F^I_{0m} = \frac{2}{3} G_{IJ} X^J F^I_{0m} = \frac{1}{3} e^{-4U} X^J \partial_m (H_I) = -\partial_m (e^{-2U}). \tag{26}$$

The Fünfbeins for the metric in (19) are

$$\begin{aligned}
e_0{}^{\underline{0}} &= e^{-2U} \quad , \quad e_m{}^{\underline{n}} = e^U \delta_m{}^n \\
e_0{}^{\underline{m}} &= e_m{}^{\underline{0}} = 0 \\
e^0{}_{\underline{0}} &= e^{2U} \quad , \quad e^m{}_{\underline{n}} = e^{-U} \delta^m_n \\
e^0{}_{\underline{m}} &= e^m{}_{\underline{0}} = 0
\end{aligned} \tag{27}$$

For the spin connections one obtains [e]

$$\omega_0^{\underline{0m}} = e^{-U} \partial_m (e^{-2U}), \qquad \omega_m^{\underline{np}} = \delta_m{}^n \partial_p U - \delta_m{}^p \partial_n U \tag{28}$$

with the rest vanishing.

Now we proceed to show that the configuration defined by (19) is supersymmetric. First we start with the time component of the gravitino variation,

$$\delta\psi_0 = D_0 \epsilon + \frac{i}{8} X_I \Gamma_0{}^{\nu\rho} F_{\mu\rho}{}^I - \frac{i}{2} \Gamma^\rho X_I F_{0\rho}{}^I \epsilon, \tag{29}$$

since the only non-vanishing component of the field strength is F_{0m}, the second term on the right hand side vanishes and the above equation becomes

$$\delta\psi_0 = \frac{1}{2} \omega_0^{\underline{0m}} \Gamma^{\underline{m}} \Gamma^{\underline{0}} \epsilon - \frac{i}{2} \Gamma^\rho X_I F_{0m}{}^I \epsilon, \tag{30}$$

then using (28) and (26) as well as the relation

$$\Gamma^m = e^{-U} \Gamma^{\underline{m}} \tag{31}$$

we obtain,

$$\delta\psi_0 = 0 \Longleftrightarrow \Gamma^{\underline{0}} \epsilon = -i\epsilon \tag{32}$$

As a consequence generically one half of supersymmetry is broken.

Next, we turn to the space component for the gravitino supersymmetry transformation law, this is given by

$$\delta\psi_m = D_m \epsilon + \frac{i}{8} X_I \Gamma_m{}^{\nu\rho} F_{\nu\rho}{}^I \epsilon - \frac{i}{2} \Gamma^0 X_I F_{0m}{}^I \epsilon, \tag{33}$$

[e] $\omega_\mu^{ab} = 2e^{\nu[a} \partial_{[\mu} e_{\nu]}{}^{b]} - e^{\rho a} e^{\sigma b} e_{\mu p} \partial_{[\rho} e_{\sigma]}{}^p$

where

$$\begin{aligned}
\mathcal{D}_m\epsilon &= \partial_m\epsilon + \frac{1}{4}\omega_m^{\underline{np}}\Gamma^{\underline{n}}\Gamma^{\underline{p}} \\
&= \partial_m\epsilon + \frac{1}{4}\partial_p U[\Gamma^{\underline{m}}, \Gamma^{\underline{p}}]\epsilon \\
\frac{i}{8}X_I\Gamma_m{}^{\nu p}F_{\nu p}{}^I\epsilon &= -\frac{i}{4}e^{2U}\partial_p(e^{-2U})\Gamma^{m0p}\epsilon = \frac{1}{4}\partial_p U[\Gamma^{\underline{p}}, \Gamma^{\underline{m}}]\epsilon \\
\frac{i}{2}X_I\Gamma^0 F_{0m}{}^I\epsilon &= \partial_m U\epsilon
\end{aligned} \tag{34}$$

where in deriving the above relations, we have used the equation satisfied by the spinor (32), our Ansatz for the gauge fields and

$$\Gamma^{m0p} = \frac{1}{2}\Gamma^0[\Gamma^p, \Gamma^m] = \frac{1}{2}\Gamma^{\underline{0}}[\Gamma^{\underline{p}}, \Gamma^{\underline{m}}]. \tag{35}$$

Substituting the above expressions into (33), one obtains a simple differential equation for the spinor ϵ

$$(\partial_m + \partial_m U)\epsilon = 0 \tag{36}$$

This gives

$$\epsilon = e^{-U}\epsilon_0 \tag{37}$$

where ϵ_0 is a constant spinor satisfying $\Gamma^{\underline{0}}\epsilon_0 = -i\epsilon_0$.

We now turn to the gauginos supersymmetry variation given in (16). Using equations (15) and (22) this can be rewritten in the form

$$\begin{aligned}
\delta\lambda_i &= -\frac{1}{4}G_{IJ}\partial_i X^I\Gamma^{\mu\nu}\epsilon F^I_{\mu\nu} + \frac{3i}{4}\Gamma^\mu\partial_\mu X_I\partial_i X^I\epsilon \\
&= \frac{1}{2}G_{IJ}\Gamma^m\Gamma^0\partial_i X^I F^J_{0m}\epsilon + \frac{3i}{4}\Gamma^m\partial_m X_I\partial_i X^I\epsilon
\end{aligned} \tag{38}$$

Using

$$\begin{aligned}
G_{IJ}F^J_{0m} &= \frac{1}{2}e^{-4U}\partial_m(H_I), \qquad \partial_m X_I\partial_i X^I = \frac{1}{3}e^{-2U}\partial_m(H_I)\partial_i X^I \\
\Gamma^0 &= e^{2U}\Gamma^{\underline{0}}, \qquad \Gamma^{\underline{0}}\epsilon = -i\epsilon,
\end{aligned}$$

it can be easily seen that the gaugino supersymmetry variation vanishes identically. Thus, we have shown that the configuration (19) defines a supersymmetric bosonic configuration, which breaks generically one half of $N = 2$ supersymmetry. We note here that supersymmetry does not restrict the functions H_I. The Bianchi identities and equations of motion for the gauge fields restrict H_I to be harmonic functions.

Let us now look at the behaviour of our solution near the horizon. First we write our solution in polar coordinates, this gives

$$ds^2 = -e^{-4U}dt^2 + e^{2U}(dr^2 + r^2 d\Omega_3^2). \tag{39}$$

Near the horizon ($r \to 0$), e^{2U} can be approximated as

$$(e^{2U})_{hor} = \frac{1}{3}(X^J H_J)_{hor} = \frac{1}{3}(X^J)_{hor}(\frac{q_J}{r^2}) = \frac{1}{3}\frac{Z_{hor}}{r^2} \tag{40}$$

where $Z = q_I X^I$ is the central charge, and Z_{hor} is its value at the horizon. The Bekenstein-Hawking entropy, $\mathbf{S}_{BH}$ related to the area of the horizon $\mathbf{A}$ is given by

$$\mathbf{S}_{BH} = \frac{\mathbf{A}}{4G_N}; \qquad \mathbf{A} = 2\pi^2(r^2 e^{2U})^{\frac{3}{2}}_{r=0} = 2\pi^2\Big(\frac{Z_{hor}}{3}\Big)^{\frac{3}{2}} \tag{41}$$

where G_N is Newton's constant.

Finally, using (40), the second equation in (19) which defines the moduli over space time, becomes near the horizon

$$(ZX_I)_{hor} = q_I, \tag{42}$$

which is the equation obtained from the extremization of the central charge [25],[18]. If one assumes that the values of the moduli at the horizon are valid throughout the entire space-time, then one obtains the double-extreme black hole solution [25] where the metric takes the Tanghelini form [21],[22]

$$ds^2 = -\Big(1 + \frac{Z}{3r^2}\Big)^{-2} dt^2 + \Big(1 + \frac{Z}{3r^2}\Big)(dr^2 + r^2 d\Omega_3^2). \tag{43}$$

4 Example: STU=1

In this section we discuss the black hole solution for the so-called STU model. This model can be obtained from the compactification of heterotic string theory on $K_3 \times S_1$ [23]. The tree-level prepotential in this model is given by $STU = 1$ and corresponds to the scalar manifold of (10) (for $n = 2$). For this case we get the following equations

$$\begin{aligned} e^{2U}TU &= H_0, \\ e^{2U}SU &= H_1, \\ e^{2U}ST &= H_2, \end{aligned} \tag{44}$$

where H_0, H_1 and H_2 are harmonic functions. ¿From (44) we obtain the following solution for the metric and the moduli fields,

$$e^{2U} = (H_0 H_1 H_2)^{\frac{1}{3}} \tag{45}$$

and

$$\begin{aligned} S &= \Big(\frac{H_1 H_2}{H_0^2}\Big)^{\frac{1}{3}}, \\ T &= \Big(\frac{H_0 H_2}{H_1^2}\Big)^{\frac{1}{3}}, \\ U &= \Big(\frac{H_0 H_1}{H_2^2}\Big)^{\frac{1}{3}}. \end{aligned} \tag{46}$$

The metric solution is given by

$$ds^2 = -(H_0H_1H_2)^{-\frac{2}{3}}dt^2 + (H_0H_1H_2)^{\frac{1}{3}}(dr^2 + r^2 d\Omega_3^2) \tag{47}$$

If the harmonic functions are represented by $H_0 = h_0 + \frac{q_0}{r^2}$, $H_1 = h_1 + \frac{q_1}{r^2}$ and $H_2 = h_2 + \frac{q_2}{r^2}$, then for an asymptotically flat metric one should demand that $h_0h_1h_2 = 1$ The ADM mass, M_{ADM}, of the black hole in five dimensions is given by[f]

$$g_{tt} = 1 - \frac{8G_N M_{ADM}}{3\pi} + \cdots \tag{48}$$

Thus for our black hole,

$$M_{ADM} = \frac{\pi}{4G_N}(\frac{q_0}{h_0} + \frac{q_1}{h_1} + \frac{q_2}{h_2}) \tag{49}$$

Eq. (46) gives the values of special coordinates at infinity, these are

$$X^0_\infty = \frac{1}{h_0}, \quad X^1_\infty = \frac{1}{h_1}, \quad X^2_\infty = \frac{1}{h_2} \tag{50}$$

and it can be easily seen that the mass of the black hole saturates the BPS bound

$$M_{ADM} = \frac{\pi}{4G_N} Z_\infty, \tag{51}$$

where Z_∞ is the value of the central charge at infinity, $Z_\infty = \frac{q_0}{h_0} + \frac{q_1}{h_1} + \frac{q_2}{h_2}$.

Moreover, from (46) one could easily see that irrespective of the values of the moduli at infinity, the values, near the horizon ($r \to 0$), (S, T, U) are always given by

$$S_{hor} = \Big(\frac{q_1q_2}{q_0^2}\Big)^{\frac{1}{3}}, \qquad T_{hor} = \Big(\frac{q_0q_2}{q_1^2}\Big)^{\frac{1}{3}}, \qquad U_{hor} = \Big(\frac{q_0q_1}{q_2^2}\Big)^{\frac{1}{3}}. \tag{52}$$

These solutions can also be derived from the stabilisation equations $ZX_I = q_I$ [18]. The central charge near the horizon is thus given by $Z_{hor} = 3(q_0q_1q_2)^{\frac{1}{3}}$.

Moreover, the Bekenstein-Hawking entropy $\mathbf{S}_{BH}$ which is defined in terms of the 3-area of the regular $r = 0$ horizon $\mathbf{A}$, is given by

$$\mathbf{S}_{BH} = \frac{\mathbf{A}}{4G_N}; \qquad \mathbf{A} = 2\pi^2\Big(\frac{Z}{3}\Big)^{\frac{3}{2}} = 2\pi^2\sqrt{q_0q_1q_2}. \tag{53}$$

Finally, the double extreme limit, where the values of the moduli are constant everywhere and are equal to their values at the horizon given by (52), corresponds to an extreme solution with the choice

$$h_0 = \Big(\frac{q_0^2}{q_1q_2}\Big)^{\frac{1}{3}}, \qquad h_1 = \Big(\frac{q_1^2}{q_0q_2}\Big)^{\frac{1}{3}}, \qquad h_1 = \Big(\frac{q_2^2}{q_0q_1}\Big)^{\frac{1}{3}} \tag{54}$$

[f] see for example [22]

and thus the metric for the double extreme solution is given by

$$ds^2 = -\Big(1 + \frac{(q_0 q_1 q_2)^{\frac{1}{3}}}{r^2}\Big)^{-2} dt^2 + \Big(1 + \frac{(q_0 q_1 q_2)^{\frac{1}{3}}}{r^2}\Big)(dr^2 + r^2 d\Omega_3^2). \tag{55}$$

For the choice, $h_1 = h_2 = h_3 = 1$, one obtains the solution

$$ds^2 = -\Big((1 + \frac{q_0}{r^2})(1 + \frac{q_1}{r^2})(1 + \frac{q_2}{r^2})\Big)^{\frac{-2}{3}} dt^2 + \Big((1 + \frac{q_0}{r^2})(1 + \frac{q_1}{r^2})(1 + \frac{q_2}{r^2})\Big)^{\frac{1}{3}} \tag{56}$$

which is equivalent to the five-dimensional black hole constructed in [29]. Clearly the entropy is still given by (53) since it is independent of the values of $h's$.

5 Conclusions

In this paper, we have constructed general BPS black hole solutions for $N = 2$, $d = 5$ supergravity coupled to an arbitrary number of vector multiplets. We have demonstrated that these solutions admit supersymmetry by solving for the supersymmetry transformation laws for the gauginos and the gravitino, making use of the underlying very special geometry structure which governs the couplings in these theories. Using our explicit solution, we have verified that the horizon acts as an attractor on which the scalar fields take constant values which extremise the central charge. In other words, the equations defining the special coordinates in our solutions reduce, near the horizon, to the stabilisation equations obtained from the extremisation of the central charge of the underlying $N = 2$, $d = 5$ supersymmetry algebra. The fixed values are independent of the initial configuration at infinity. The dependence of the entropy on the central charge is also derived. As an example we considered the STU model in some detail and derived various physical quantities characterising its black hole solution.

Recently, it became obvious that BPS black holes play a major role in the analysis of phase transitions among $N = 2$ strings vacua. Generically, these topological phase transitions occur at points in the moduli spaces where non-perturbative BPS states become massless. For example, in $N = 2$ type IIB string vacua on Calabi-Yau three-folds, electrically or magnetically charged black holes become massless at the conifold points in the Calabi-Yau moduli spaces, where certain homology cycles of the Calabi-Yau spaces shrink to zero size [33]. Non-trivial black hole solutions provide the framework in which one can find those points in space-time where the internal Calabi-Yau periods shrink to zero size and where the transitions due to massless black holes take place [24].

In Calabi-Yau compactification of type IIA, due to the presence of theta angles, the vector moduli space can be described in terms of a complexified Kähler cone [16]. As such, as one approaches the boundary of the Kähler cone, one can go smoothly past the singularity to another phase of the conformal field theory [16]. The new phase might correspond to a new Kähler cone of a different Calabi-Yau, thus signalling a topology changing process,

or it might be related to a non-geometrical phase, *i.e* an abstract formulation of the conformal field theory, such as Landau-Ginzburg model.

In five dimensional $N = 2$ supergravity theories corresponding to M-theory compactification on a Calabi-Yau threefold, there are no axionic fields and the moduli, the sizes of 2-cycles in the Calabi-Yau, take values in a real Kähler cone. In contrast to the four-dimensional case, to go from one phase to another one has to go through the singularity. As a result, in five dimensions one gets sharp phase transition [17]. One example of such transitions is the "flop transition", where one of the moduli approaches zero and subsequently blown up, this corresponds to going into a new Kähler cone of a birationally equivalent Calabi-Yau with different intersection numbers. In analysing these flop transitions, BPS black holes which become massless at the phase transition point play a significant role [17]. Recently, the BPS central charge Z, as well as the magnetic charge defining the tension of the five-dimensional string have been employed in probing the flop phase transitions in five dimensions [18]. It would be of interest to study these transitions using our explicit black hole solutions along the lines of [24]. This is currently under investigation.

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